Math 2374 Syllabus

- How to get started
- Assessment: quizzes, Mathematica notebooks, 3 midterm exams, final exam.
- Put the exam dates on your calendar.
- No homework to hand in. Quizzes and exams are based on the homework.
- Missing exams and quizzes
- Course content is organized around Modules
- Mathematica notebooks, downloading Mathematica. Get to work on the notebooks before the session on Thursday.

There is too much to read!

Section 1.3: Matrices, determinants and the cross product.

We learn

- Determinants of 2x2 and 3x3 matrices (nxn in section 1.5)

$$\operatorname{Det}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\operatorname{Det}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 1\cdot 4 - 2\cdot 3 = -2$$

$$\operatorname{Det}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = aei + bfg + cdh - gec - hfa - idb$$

$$\operatorname{Det}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = 1 + 0 - 2 - 1 - 0 - 0 = -2$$

Determinants satisfy:
Linearity in each row and in each column
If we interchange two rows (or two columns) the determinant is multiplied by -1.
If we add a multiple of one row to another, the determinant is unchanged. (Same for columns).
If two rows (or two columns) are the same, the determinant is 0

switch 2 rows

$$\operatorname{Det}\begin{bmatrix} c & d \\ a & b \end{bmatrix} = cb - ad = -(ad - bc)$$

$$\operatorname{Det}\begin{bmatrix} a+e & b+2f \\ c & d \end{bmatrix} = \operatorname{Det}\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \operatorname{Det}\begin{bmatrix} e & 2f \\ c & d \end{bmatrix}$$

$$\operatorname{Det}\begin{bmatrix} a & b \\ a & b \end{bmatrix} = ab - ab = 0$$

$$= \operatorname{Det}\begin{bmatrix} a-a & b-b \\ a & b \end{bmatrix} = \operatorname{Det}\begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix} = 0$$

The cross product for vectors in $\mathbb{R}^3$

Definition: If $a = (a_1, a_2, a_3)$, $b = (b_1, b_2, b_3)$

then "a cross b" is

$$a \times b = (a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1)$$

$$1 \to 2 \to 3 \to 1$$

$$= \operatorname{Det}\begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \qquad \begin{aligned} i &= (1,0,0) \\ j &= (0,1,0) \\ k &= (0,0,1) \end{aligned}$$

Properties of the cross product:

a x b is perpendicular to a and to b, and has length ||a|| ||b|| |sin u |

a , b , a x b is a right-handed set of vectors.

Pre-Class Warm-Up !!

- Evaluate the cross product

(1, -1, 2) x (-1, 2, 1)

Answer:

a. (5, 3, -1)

b. (-1, -2, 2)

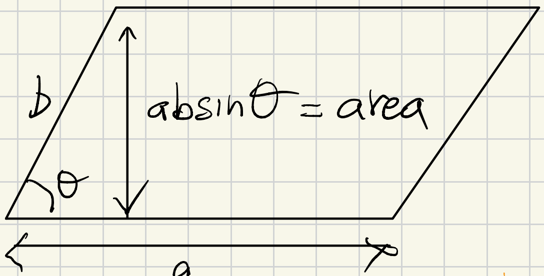
c. (-5, -3, 1)

d. (1, -2, -2)

e. None of the above

Determinants, cross products, areas of parallelograms and volumes of parallelepipeds

In 3-D, a x b has length equal to the area of the parallelogram determined by a and b.

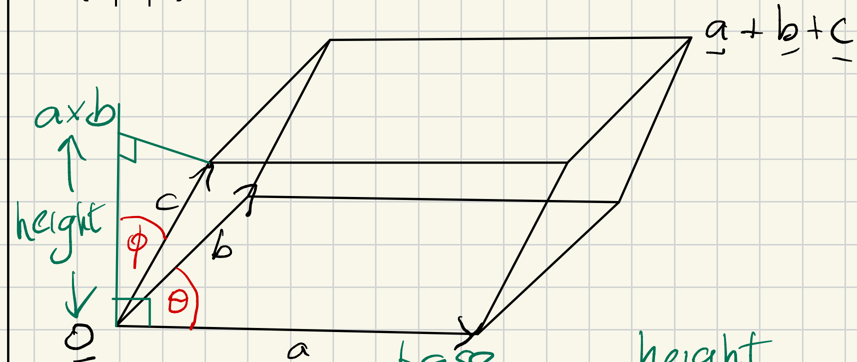


$$(a \times b)\cdot c = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \cdot c$$

$$= \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

In 3-D the volume of the parallel~~o~~epiped determined by a , b and c is the triple scalar product (a x b) • c (in absolute value). Why?

This equals the determinant of the matrix {a|b|c}. Why?



$$\text{Volume} = \overbrace{\|a\|\|b\||\sin\theta|}^{\text{base}}\ \overbrace{\|c\||\cos\phi|}^{\text{height}}$$

$$= \text{base} \cdot \text{height}$$

$$= \|a \times b\| \ \|c\| \ |\cos\phi|$$

$$= |(a \times b)\cdot c|$$

Why does the triple product $(a \times b) \bullet c$ equal Det $\{a|b|c\}$?

This means we don't really need the triple product.

Summary of parallelepipeds:

In every dimension the volume of the parallelepiped determined by n vectors equals the absolute value of the determinant of the matrix they form.

Dimension 3: we have seen it, and it's on page 40

Dimension 2: it's deduced on page 39

Also we have seen that the area of the parallelogram determined by 2 vectors in 3-D is the length of the cross product.

Equations of planes, distance from a point to a plane, line of intersection of two planes, etc.

You have HW questions like:

Find an equation of the plane that is perpendicular to $(1,2,3)$ and passes through $(1,-1,0)$

It is $1\cdot x+2\cdot y+3z=D$. Find D

$1\cdot 1+2(-1)+3\cdot 0=D=-1$

Find the distance from the point to the plane

Find the intersection of the two planes

See the second video.

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The equation of a plane in 3-space has form

$Ax + By + Cz + D = 0$

or $Ax + By + Cz = D$

Why?

$Ax + By + Cz = 0$ is the plane perpendicular to the vector (A,B,C) passing through $\underline{0}$.

This equation says

$(A,B,C)\cdot(x,y,z)=0$,

(A,B,C) is perp to (x,y,z)

What about D?

Example: Find the equation of the plane perpendicular to $(1,-2,3)$ passing through $(-1,0,2)$.

Possible answers:

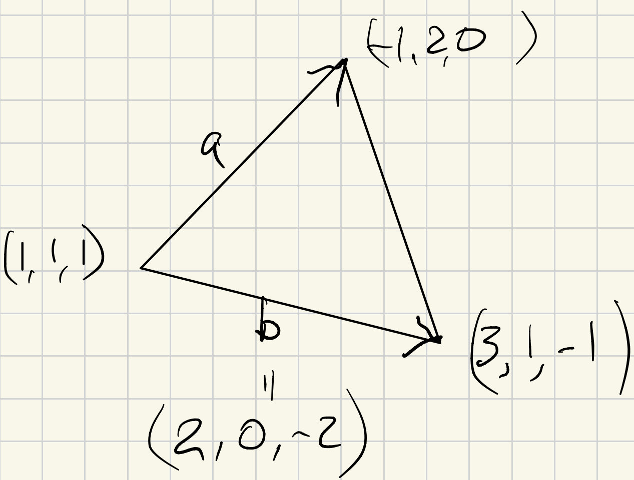
1. $-x + 2z = 2$
2. $x - 2y + 3z = 5$ ✓
3. $x - 2y + 3z = 1$
4. $x - y + 2z = 3$
5. None of the above -

$$1 \cdot x - 2y + 3z =$$

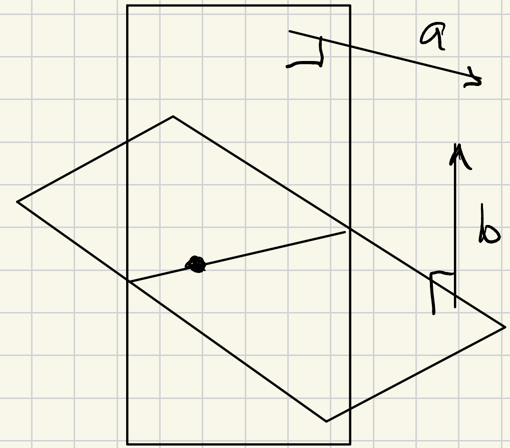
$$1(-1) - 2 \cdot 0 + 3 \cdot 2 = 5$$

Like page 42 Example 11

Find the equation of the plane passing through the points
(1,1,1), (-1,2,0) and (3,1, -1)



$a \times b$ is perp. to the plane.



$a \times b$ points in the direction of the line.

Like page 50 qn 20: Find the intersection of two planes.

Like page 43

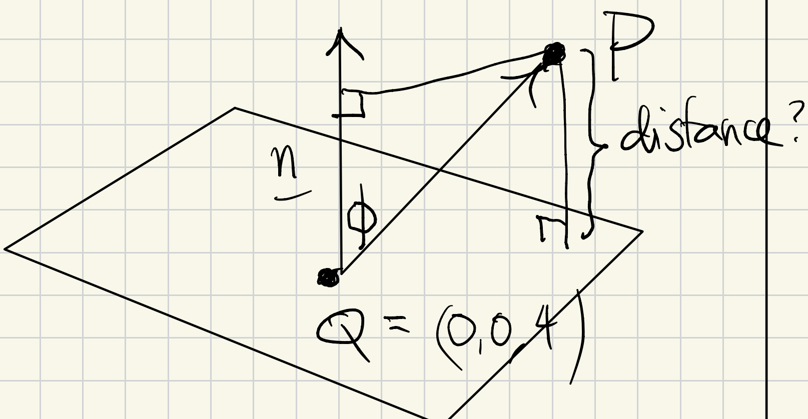
Find the distance from the point (1,0,-1) to the plane $2x - y + z = 4$

First: which of the following points lie in the plane?

1. (1,0,-1)
2. (1,0,2) ✓
3. (1,1,1)

(0,0,4) lies in plane.

Could you find another point in the plane?



$$\underset{\sim}{n} \cdot (P - Q) = \|n\| \, \|P - Q\| \cos\phi$$

$$\text{distance} = \|P - Q\| \cos\phi$$

$$= \frac{\underset{\sim}{n} \cdot (P - Q)}{\|n\|}$$